Set theory - Winter semester 2016-17

Problems	Prof. Peter Koepke
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Problem 19 (3 points)**.** Show that for any ordinal α, $\alpha+\omega$ is a limit ordinal. Use this to show that the class of all limit ordinals is a proper class.

Problem 20 (3 points)**.** The *Collection Scheme* states that for every relation R and for every set x, there is a set y such that

$$\forall u \in x\ (\exists v\ (u,v) \in R \Rightarrow \exists w \in y\ (u,w) \in R).$$

Prove the Collection Scheme *(hint: use the von Neumann hierarchy.)*

Problem 21 (6 points)**.** Prove that the definitions of the following functions and relations on $\mathbb{Q}$ in Definition 53 d)-f) are independent of the representatives.

(1) $+^{\mathbb{Q}}$
(2) $\cdot^{\mathbb{Q}}$
(3) $<_{\mathbb{Q}}$

Problem 22 (4 points)**.** Show that $(\mathbb{R}^+, \cdot^{\mathbb{R}}, 1^{\mathbb{R}})$ is a multiplicative group.

Problem 23 (4 points)**.** Show that $V_{\omega+\omega}$ is closed under the following operations.

(1) Power sets, pairs, ordered pairs and unions.
(2) For every formula $\varphi(x,y,z)$, the map f_φ sending an ordered pair (y,z) to the set

$$f_\varphi(y,z) = \{x \in y \mid \varphi(x,y,z)\}.$$

Due Friday, November 18, before the lecture.